PS Algorithms and Data Structures 2024

Task sheet 7

Task 19

It is given a hash table of size m and a natural number n. To insert the elements from the universe U with $|U| \geq n \cdot m$ an arbitrary hash function $h : U \to \{0, 1, \ldots, m-1\}$ is used.

Show that U has a subset T of size n for which all elements collide with each other.

Task 20

Given a graph G in a) adjacency list representation and b) matrix representation. Answer the following questions for both representations.

1. What (tight) runtime complexity does the calculation of the out-degree of a node have?
2. What (tight) runtime complexity does the calculation of the in-degree of a node have?

Specify the complexities using Θ notation based on the parameters out-degree, in-degree, number of nodes, and number of edges.

Task 21

Two nodes u and v in a graph are called *connected* if in the graph (1) there is a path from u to v and (2) a path from v to u. In undirected graphs, statements (1) and (2) are equivalent.

Develop an algorithm with runtime $O(|V| + |E|)$ that assigns an (integer) label $v.\ell$ to each vertex v of a given undirected graph $G = (V, E)$ such that $v.\ell = u.\ell$ for every pair of vertices u and v if and only if u and v are connected.