PS Algorithms and Data Structures 2024

Task sheet 11

Task 31

Develop a divide-and-conquer algorithm that calculates the height of a binary search tree with n nodes and show that its running time is $O(n)$.

Hint: Set up a recursive runtime estimate and solve it directly using a suitable induction hypothesis (instead of trying to apply the master theorem).

Task 32

A matching M of an undirected graph is a subset of the edges for which it holds that each node is incident to at most one edge from M. A maximal matching (also called non-extensible matching) is a matching to which no more edges can be added without violating the above property.

Develop an algorithm with runtime $O(|V| + |E|)$ that calculates a maximal matching of an undirected graph $G = (V, E)$.

Task 33

Jobs n are processed on a single processor. Each job i has a duration t_i and a deadline d_i by which it is to be completed.

We consider the scheduling problem of creating a processing sequence for the jobs so that all jobs are completed by their respective deadline if possible. If this is not possible, individual jobs can be completed late. A job i is completed late if $f_i > d_i$ applies to its completion time f_i. This job then has a delay $\ell_i = f_i - d_i$. If job i is completed on time, then $\ell_i = 0$ is set. The aim of creating the processing sequence is to minimize the sum of all delays.

Show that none of the following greedy strategies provides an optimal solution:

1. *Job Length*: The jobs are sorted by ascending duration t_i.
2. *Earliest Deadline*: The jobs are sorted by ascending deadline d_i.
3. *Slack Time*: The jobs are sorted by ascending slack time $d_i - t_i$.