

# PS Algorithms and Data Structures 2026

## Task sheet 8

### Task 22

Let  $G = (V, E)$  be an undirected, unweighted graph. The eccentricity of a vertex  $u$  is defined as  $Ecc(u) = \max_{v \in V} dist(u, v)$ . Furthermore, the radius of a graph is defined as the minimum eccentricity, i.e.,  $R(G) = \min_{u \in V} Ecc(u)$ .

Formulate an algorithm that computes  $R(G)$ . What is the runtime complexity of your algorithm?

### Task 23

Rewrite the depth-first search so that a stack is used instead of recursion.

Make sure that your approach sets the entries  $u.d$  and  $u.f$  of each node  $u$  to the same values as the depth-first search presented in the VO.

### Task 24

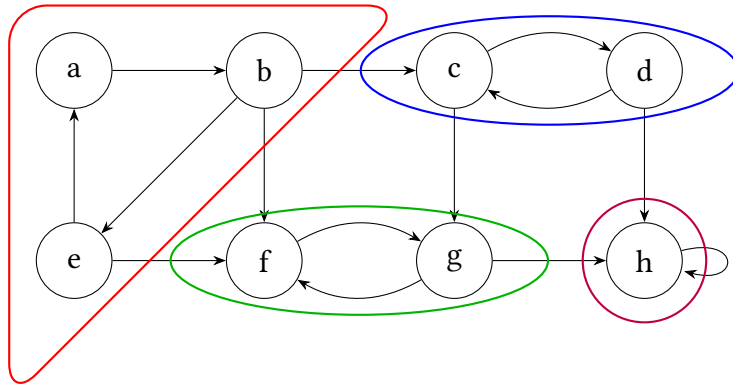
Let  $G = (V, E)$  be a directed graph.

A strongly connected component  $C$  of a graph  $G$  is a maximal set of vertices  $C \subseteq V$  such that for every pair of vertices  $u, v \in C$  it holds that  $u$  and  $v$  are mutually reachable. That is, there is both a path from  $u$  to  $v$  and a path from  $v$  to  $u$ . Figure ?? shows an example of a graph with four strongly connected components.

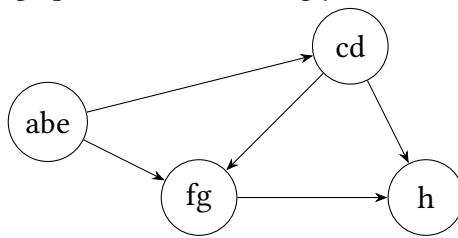
Let  $C_1, C_2, \dots, C_k$  be the strongly connected components of  $G$ . We construct the component graph  $G^{SCC} = (V^{SCC}, E^{SCC})$  as follows.  $V^{SCC} = \{v_1, \dots, v_k\}$  contains one vertex for each component. There is an edge  $(v_i, v_j) \in E^{SCC}$  if and only if there exists a directed edge  $(x, y) \in E$  with  $x \in C_i$  and  $y \in C_j$ . Figure ?? shows an example of a component graph. The vertices of each strongly connected component have been merged. For example,  $a, b$ , and  $e$  were merged into the new vertex  $abe$ .

We define the transposed graph  $G^T = (V, E^T)$  with  $E^T = \{(u, v) \mid (v, u) \in E\}$ . Thus  $G^T$  has all edges of  $G$  reversed. Figure ?? shows the transposed graph of our example.

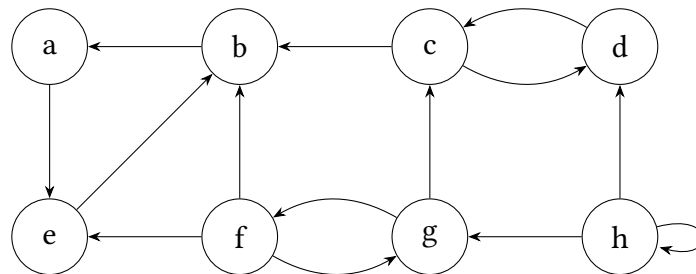
Show that  $((G^T)^{SCC})^T = G^{SCC}$ , i.e., the transpose of the component graph of  $G^T$  is the component graph of  $G$  (in general, not only for the example).



(a) A directed graph  $G$  with four strongly connected components.



(b) The component graph  $G^{SCC}$  of  $G$ .



(c) The transposed graph  $G^T$ .

Figure 1