PS Algorithms and Data Structures 2026

Task sheet 10

Aufgabe 28

Develop an algorithm with runtime complexity $O(|V|^{2.81})$ that determines whether a given undirected, unweighted graph $G = (V, E)$ contains a 4-cycle. Remember that (except for the start and end node) no node appears multiple times inside a cycle.

Aufgabe 29

An independent set S of an undirected graph is a subset of the vertices such that for every pair $u, v \in S$, u and v are not connected, i.e., there is no edge $\{u, v\}$. A maximal independent set is an independent set to which no further vertex can be added without violating the above property.

Develop an algorithm with runtime complexity $O(|V| + |E|)$ that computes a maximal independent set of an undirected graph $G = (V, E)$.

Aufgabe 30

The Floyd-Warshall algorithm computes the shortest distances between all pairs of vertices (APSP) and has a runtime complexity of $O(|V|^3)$. Suppose we have already computed all shortest paths using Floyd-Warshall and stored them in the matrix $D^{(n)}$. Now, we update the graph by inserting either a new edge or a new vertex. This can generally change the shortest paths. For both cases of

1. insertions of an edge between existing nodes and
2. insertions of a vertex with incident edges

describe an algorithm for updating the result. What are the runtime complexities of your algorithms? (The runtime complexities should of course be lower than $O(|V|^3)$ and thus asymptotically better than a recalculation using Floyd and Warshall's algorithm.)